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The Role of Production Control Using Linear Decision Rules in Production and Operations Management

Ageng'o Jim Roberts [a]*, Tibbs Charles [a], Cheruiyot Joseph [a] and Soy Wilson Kiptanui[b]

[a]Kabianga University College, P.O Box 1-20200, Kabianga, Kericho.
[b]James Finlay (k)Ltd, P.O.Box 223-20200, Kericho. Email: agengojimroberts@yahoo.com

*Author for correspondence and reprint requests

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The fluctuations in orders from customers create difficulty for managers who are responsible for scheduling production and employment. Changes in shipments must be absorbed by some combination of the following actions. These include adjusting the size of the work force, adjusting the finished goods inventory, adjusting the amount of overtime work, and finally adjusting the order backlog. All these actions are related and since each of them has certain associated costs, one of the prime responsibilities of production management is to make decision that represent minimum cost choices. It is very difficult to predict the fluctuations in orders and that of decision-making assignment is even more complex in the common circumstances of unforeseen changes in demand. However, the importance of the fundamental responsibility is clear. Good decisions within a company contribute directly to its profits. "It is also known that better decisions in many companies can increase the efficiency with which the nation uses its resources---a fact of growing importance as our arms race with the Soviet Union intensifies.

To understand the concepts of this work better, an account of the report of some findings of a research team that has been studying the application of mathematical techniques to the scheduling of production and employment is given. As a result of this work, new methods have been developed for improving the quality of scheduling decisions and for helping managers to make substantially better decisions than they could make by using prevailing rule-of-thumb and judgment procedures. Once a general rule has been developed, the computations required to establish a monthly production schedule can be completed by an employee in a few hours or by a computer in just a few minutes. The new techniques may be illustrated by a comparison of the actual performance of the factory under management's scheduling decisions with the performance that would have been realized if the new technique has been used; it indicated a cost advantage of at least 8.5 percent for the mathematical decision rule, with further gains to be derived from improved sales forecasting. The plant in question was not a large one; there were only one hundred employees. Yet the annual saving amounted to approximately Kshs. 4 million, reflecting reductions in a number of cost items, including regular, payroll, overtime, hiring, training, layoff and inventory.

Key words: Linear decision rules and production management.

Introduction

Production and Mathematics

Most managers in many firms have adapted the use of mathematics in production because they think that with more knowledge in that field there will be more accuracy in the predictions of production. To use mathematics as a tool, one must understand it as a language. Since it differs from the language of production, the essential first step in applying it to a plant problem is to translate the description of production from its familiar vocabulary into the language of mathematics. In most cases, if not all, such transformation calls for generalizing, quantifying, and identifying the goals and constraints (*limitations*)

Elements of Decisions

As far as production process is concerned, orders (on hand or anticipated) generate production. At the other end of the process, shipments satisfy orders. Within these limits the process accumulates costs. Total costs for a given time period are influenced by managerial scheduling decisions. “These decisions commonly are taken with reference to selected goals. Certain costs, for example, are associated with stability of the production schedule over time”. (C.C. Holt, F. Modigliani, and J.F. Muth, 1963)

If there is steady employment of a group of workers, costs are lower than if the group fluctuates in size. Costs associated with hiring, training, layoffs, and overtime are minimized, as well as tangible costs related to under time operations. If incoming orders are not stable, however, a level rate of production can be maintained only by accepting fluctuations in the backlog or by making shipments as required from a buffer stock of finished goods. :Any decisions to absorb fluctuations through finished-goods inventory commits the firm to direct investment costs and to the expenses associated with storage, handling, spoilage, obsolescence, and adverse price changes”. H. A. Simon, C. C. Holt, and F. Modigliani, 1964. Similarly a decision to absorb fluctuations through a buffer backlog also has recognizable costs associated with it, although these are not measured by standard accounting techniques --- the costs of customer dissatisfactions, loss of future business, and adverse price changes.

Generally, in most settings, production decisions are further complicated by the movement of several products through common facilities and work groups. Another common problem which is associated with and is often encountered is the variable procurement costs for materials and parts, which are related to purchase lot size and stability of incoming deliveries. Finally, decisions strategy must also consider the effect of errors in forecasting future orders and of the accumulation of scheduling decisions over successive time intervals. Both these considerations compel the adoption of dynamic strategy that combines a judgment as to the impact of the orders-stock-production-shipment complex on the immediately upcoming time period with a judgment designed to compensate for prior errors in scheduling preceding time periods.

Results and Discussions

Obtaining optimum Results

The best, or minimum –cost, decision in this complicated setting with its multitude of interrelated variables is far from obvious. There is no easy way out that can be utilized. One management may decide to pursue a shifting strategy outline by rule-of-thumb procedures; another management may decide to adopt a stable strategy designed to realize a single objective, such as level employment or prompt delivery of customers' orders. However, little argument is needed to show that such strategies cannot, except in extraordinary circumstances, produce optimum results.

Since every fluctuation in incoming orders can be met only by a choice of alternatives, each of which carries an inescapable set of associated costs, the scheduler is confronted with a complex and dynamic situations in which optimum performance requires absorption

Managers must agree on the objective of maximizing or minimizing a specific criterion. For the firm as a whole, this criterion would be profits, and for the production scheduling manager who controls neither sales nor profits, the critical criterion would be minimizing the costs of operations. All costs must be described quantatively in comparable units, including intangible costs and those not regularly identified by financial and cost-accounting systems. A reporting and planning period must be selected for the accumulation and analysis of information relevant to scheduling decisions. The selection of the decision period itself is a problem. The significant factors include the size of errors in forecasting incoming orders, the cost of making the forecast, the time required to gather new information to improve earlier forecasts, the cost of making and administering decisions, and the relative costs of making a large number of small scheduling changes and a small number of large ones.

The process of quantifying intangibles, such as the costs associated with maintaining a buffer backlog of accepted but unproduced orders, is a process of making numerically explicit certain values that are always present in Management thinking, but in an ill-defined and cloudy form. Looking into it, we can see that actual precision in doing this is neither possible nor necessary. But it is essential to assign numerical weights to all variables and to recognize that doing this is no more than translating from language that permits the implicit to a language that compels the explicit.

Further, the general decision problem must be expressed in a mathematical form that is flexible enough to comprehend the full range of production costs and simple enough to permit relatively easy solution. If we consider the nature of the costs associated with production, as outlined above, we will find that a U-shaped curve is useful for general expression. This step would again be justified by the example given below.

High costs are incurred in holding both large inventories and inventories so small that out-of-stock conditions are common, with consequent delays in shipments, short production runs to fill back orders, and customer dissatisfaction.

Similarly, frequent scheduling of both overtime and undertime (a less-than-full employed work force) is expensive.

Such costs are often intangible and are not explicitly reported in accounting procedures, but they must be explicitly quantified for mathematical treatment. Somewhere between the extremes of overtime and undertime, labor costs are at a minimum. These considerations indicate the feasibility of achieving a reasonable and workable approximation of the complex of production costs by the simplest mathematical expression that gives a U-shaped curve and a quadratic function.

Assumptions and Advantages

By this time, there should be an idea that the view of the problem does not assume that the costs of hiring workers equal the costs of laying them off, or that changes in costs in either direction are symmetrical. It does not assume that the costs associated with adding to inventory holdings equal the costs of depleting the inventory or that change in either direction are symmetrical.

There is always one common misunderstanding about the language of mathematics---the belief that precise numerical expression requires equal precision in reporting ‘facts.’ Mathematics can be an effective decision-making tool even in circumstances in which the values assigned to costs represent no more than approximations.”[5] In this sense, the mathematical approach is more precise and consistent, and therefore more rational than judgment based on experience and an informed “hunch.” It compels the scheduler to consider all criteria previously defined as essential, and it also compels him to consider them consistently every time a scheduling decision is made. In fact, after the decision rule, expressed as a formula with explicit values for specified constant elements, has been framed, it does the considering for the scheduler as a routine of the mathematical process.

It is true that the ultimate judgment of the efficacy of a mathematical decision tool is better decisions, and that operations are scheduled at lower costs that decisions arrived at by alternative methods. This can be demonstrated by matching the actual record under established scheduling procedures with the record that would have been made under the mathematical decision rule.

To test the application of the general mathematical techniques described above, the research study conducted by a research team which studied the scheduling problem in a paint factory would be given to illustrate the test. To simplify the analysis, without changing its fundamental concept, scheduling decisions were assumed to be made monthly and costs were accumulated over the same period.

Cost Components (Procedure)

First, the following kinds of cost components were identified; regular payroll, hiring and layoff, overtime and under time, inventory, back order and machine setup

All these costs were developed as discrete components and then were combined in an expression of the complete cost function for the factory as a whole.

Payroll

With monthly adjustments in the size of the work force, regular payroll costs per month were a linear function of the size of the work force; that is, if they had been correlated on a graph, with payroll costs on the vertical axis and work-force size on the horizontal axis, the resultant diagonal line would have been fairly straight. Payroll dollars for regular work time also varied directly with the size of the work force measured in man-months.

Genearally, hiring and layoff costs were associated with the magnitude of change in the size of the force. Costs of hiring and training rise with the number of workers hired and trained; layoffs are associated with the number of workers discharged. We can therefore see that there is no necessary symmetry between hiring and layoff costs in their relation to the number of workers processed; and random factors reflecting the tightness of the local labor market or reorganization of the work structure at certain levels of employment may also be present sporadically. The representation of these costs by u-shaped curve, therefore, was only an approximation of the average costs of changes of various magnitudes in the work force.

Overtime

Overtime operations in the factory involved wage payments at an hourly rate of 50 percent higher than the regular time rate. Undertime costs, reported only indirectly through the accounting system, and reflected waste of labor time measured by the difference between the actual monthly wage bill and the wage bill for the smaller work force that would have sufficed to accomplish the actual production. Actual overtime during any month is determined, of course, not only by a work load in excess of that which can be produced by the regular force in regular hours, but also by such random disturbances as emergency orders, machine breakdown, quality control problems, fluctuations in productivity, and any other occurrences of the same nature that can cause disturbances.

In setting the production rate and the work force for the month, the scheduler must balance the risk of maintaining too large a work force against the risk of holding a smaller work force but being required to pay overtime compensation. As in the case of hiring and layoff costs, these considerations suggested a u-shaped, possibly unsymmetrical, curve.

Inventory

Absorbing order fluctuations through inventory and back order buffer gives rise to new costs. Holding a good size inventory incurs costs such as interest, obsolescence, handling, storage, and adverse price movements. On the other hand, a decision to reduce these costs by operating with a small inventory invites out-of-stock conditions with the associated costs of delayed shipments, lost sales, and added machine setups for special production runs to balance out stocks and to service mandatory shipments. The analysis pointed to the need for an optimum inventory level at which combined costs were at a minimum.

Cost Function Developed

The complete cost function for production and employment scheduling was developed by adding the components reviewed by the study team when it had finished studying the paint company case. The mathematical generalization was then applied to specific situations in the paint factory by inserting numerical values representing estimates of various costs involved.
From the estimates, it can be seen that some were drawn directly from accounting data or obtained through statistical treatment of accounting data to complete the study of the paint factory. Other estimates, such as those for intangible costs of delayed shipments, were subjective. Here it is important to note that the accuracy of the estimates was not a critical consideration. An analysis of the effect of errors as large as a factor of two--that is, overestimating specific cost element by 100 percent, or underestimating them by 50 percent--indicated that the use of the resultant decision rules would incur costs only 11 percent higher than with the correct estimates of costs.

Decision Rules

At this point, the mathematical process led to the development of two monthly decision rules, one to set the aggregate rate of production and the other to establish the size of the work force. The two rules demonstrated by the exhibit given by the study team about the paint factory
(Exhibit I).

The production rule incorporates a weighted average of the forecasts' of twelve months' future orders, which contributes to smoothing production. The weight assigned to future orders declines rapidly because it is a lot more economical to produce for distant shipment in view of the cumulative costs of holding inventory. This accounts for the negative number for the last seven months in the production rule in Exhibit I. The employment rule also incorporates a weighted average of forecasts of future orders, with the weights rejected further into the future becoming negligible.

EXHIBIT I

Production and employment decision rules for Paint Factory

$$P_t = \begin{array}{lll} (+ & .463 & O_t\ ) \\ (+ & .234 & O_t+1\ ) \\ (+ & .111 & O_t+2\ ) \\ (+ & .046 & O_t+3\ ) \\ (+ & .013 & O_t+4\ ) \\ (- & .002 & O_t+5\ ) \\ (- & .008 & O_t+6\ ) \\ (- & .010 & O_t+7\ ) \\ (- & .009 & O_t+8\ ) \\ (- & .008 & O_t+9\ ) \end{array} + .993\, W_{t\ -1} + 153 - .464\, I_{t\ -}$$

(- .007 O_t +10)
(- .005 O_t +11)

(+ .0101 O_t)
(+ .0088 O_t +1)
(+ .0071 O_t + 2)
(+ .0054 O_t +3)
(+ .0042 O_t +4)
(- .0031 O_t +5)
Wt = .743Wt - 1 + 2.09 - .0101 (- .0023 O_t +6)
(- .0016 O_t +7)
(- .0012 O_t +8)
(- .0009 O_t + 9)
(- .0006 O_t +10)
(- .0005 O_t +11)

Where:

P_t. is the number of units of product that should be produced during the forthcoming month, t

W_{t-1} is the number of employees in the work force at the beginning of the month (end of the previous month)

I_{t-1} is the number of units of inventory minus the number of units on back order at the beginning of the month

W_t is the number of employees that will be required for the current month, t. The number of employees that should be hired is therefore $W_{t-} W_{t-1}$

O_t is the forecast of number of units of product that will be ordered for shipment during the current month, t

O_{t-1} is the same for the next month, t + 1; and so forth weights rejected further into the future becoming negligible.

The second term of the production equation (.993 W_{t-1}) as given above reflects the influence of the number of workers employed at the end of the preceding month. Because both large decreases in the payroll and large amounts of unused labor are costly, the level of scheduled production responds to the size of the work force at the start of the month.

The next two terms in the production decision rule (513 - .464 I_{t-1}) relate the inventory to production. If net inventory at the end of the preceding month is large, the negative term will exceed the positive term, with resultant downward influence on scheduled production. A reverse relationship would contribute to establishing a higher level of production. This term also functions to take account of past forecast errors, since their effect is to raise the net inventory above, or push below, the desired level.

The first term of the employment rule (.743 $W_{t-1)}$ provides for a direct influence between the work force on hand at the beginning of a month and the scheduled employment during the month, reflecting the costs associated with changing the size of the work force. The next two terms (2.09 -.010 I_{t-1}) make provision for the effect of the net inventory position on the employment decision. A large net inventory will lead to a decrease in the scheduled work force, and a small net inventory will have the opposite effect.

Both terms of the two rules make explicit the dynamic interaction of production and employment. For example, production during a month affects the inventory position at the end of the month. This affects employment decisions in the next month, which then influences the production decision in the third month. Again, the influence of the net inventory on both production and employment decisions provides a self-correcting force which operates to return inventory to its optimum position regardless of the accuracy of sales forecasts.

It is most difficult, if not impossible, to account for this interaction without a mathematical decision rule. The manager who makes these decisions on the basis of intuition and experience may hit the right answer some of the time, but he will not do so consistently.

The weighting of the sales forecasts and the feedback factors determines the magnitude of production and employment responses to fluctuations in orders, thereby allocating the fluctuations among the work force, overtime, inventory, and backlog in the interest of minimizing total costs. While the work force responds to rather long-run fluctuations in orders, the principal response of production is to near-term orders, and to the inventory position. Thus, the rule provides for the absorption of short-run fluctuations in orders and errors of forecasting by scheduling overtime operations.

The Superiority of Decision Rules

How much are decision rules of the kind described an improvement over the usual methods of scheduling production?

This question was answered for the paint factory by making a hypothetical application to scheduling in the plant and comparing the results with actual performance under established procedures. Production and employment decisions in the paint factory were analyzed for a six-year period. The production and employment decision rules were then applied to stimulate the decision that would have been made if they had been in use during the same six-year period.

Because the same data were used by the research team as by management, hindsight could be of no use to counteract it. A necessary ingredient for the comparison was a monthly series of forecasts of future orders throughout the period under analysis. And because no such forecasts had actually been recorded, the comparison could not be made on the basis of forecasts identical to those implicitly in the minds of management when it made its scheduling decisions. As a substitute, two sets of forecasts were devised which bracketed the forecasts actually used by management.

The first set of forecasts consisted of actual orders received. This was, in other words, a “perfect” forecast, assuming the future to be known in the present; use of it established an upper limit for performance.The second set of forecasts was devised by assuming at the future orders would be predicted by using a moving average of past orders. Specifically, orders for a year ahead were forecast as equal to those actually received in the preceding year. This annual forecast was then converted to a monthly forecast by applying a seasonal adjustment based on actual past performance.

A comparison of actual costs under management scheduling with hypothetical costs under the decision rules did not tell the whole story. The figures were not solid; problems of allocating costs between paint and other products processed in the plant, as well as the absence of a firm accounting underpinning for certain intangible costs, gave a tentative quality to the data. The research team judged, however, that the comparison was a valid one for all practical purposes and that cost differences shown in **Exhibit II** were highly significant. The figures cover two periods. The longest period for which cost figures were available for a three-way comparison between actual performance and expected performance under the new rules using both a perfect forecast and a moving average forecast, 1993 – 1997. The period in which company performance was matched against the decision rule using a moving average forecast 1994 – 1998.

Exhibit II shows that the general effect of he decision rules with either moving average or perfect forecast, was to smooth the very sharp month-to-month fluctuations in both production and size of work force in actual factory performance. Overtime and inventory-holding costs were somewhat higher under the rules, with the moving average forecast (a backward-looking forecast) than the actual costs were, but this excess was more than offset by the fact that back orders were consistently held at lower levels. It is worth observing that the cost associated with back orders are particularly difficult to include as a significant factor in rule-of-thumb and judgment decision.

According to exhibit II, the decision with the moving average forecast saved Kshs. 13,840,000 annually against factory performance. For this stage in the history of this plant, greater savings could have been secured by making optimum use of crude forecast than by improving forecasts. We should note that the decision rule with perfect forecast had lower costs than the same rule with the moving average forecast in the 1993-1997 period, by about 10 percent or an average of Kshs. 4,720,000 annually. The difference, which is entirely attributable to better fore- casting, is a sizable one but only about one third as large as the other saving.

In the 1994-1998 period, actual factory costs exceeded cost under the decision rule by 8.5 percent, or Kshs. 4,080,000 per year on the average. The economics of the decision rule were achieved by reducing payroll costs more than overtime costs increased, reducing back order penalty costs more than inventory holding costs increased and reducing hiring and layoff costs

EXHIBIT II
Actual performance vs. expected performance

Under Decision Rules
(in thousands of Shillings)

Costs	Company Performance	Decision Rule: Moving average Forecast	Decision Rule: Perfect Forecast
A. Cost Comparison for 1994 -1998			
Regular payroll	Kshs. 1,940	Kshs. 1,834	Kshs. 1,888
Overtime	196	296	167
Inventory	361	451	454
Back orders	1,566	616	400
Hiring & layoff	22	25	20
Total Cost	**Kshs. 4,085**	**Kshs. 3,222**	**Kshs. 2,929**
	139%	110%	100%
B. Cost comparison for 1998-2002			
Regular payroll	Kshs. 1,256	Kshs. 1,149	
Overtime	82	95	
Inventory	273	298	
Back orders	326	246	
Hiring & layoff	16	12	
Total Cost	**1,953**	**1,800**	
	108.5%	100%	

Conclusions

While further exploration of the problems involved in applying mathematical decision rules to production and scheduling decisions seems clearly desirable as a basis for definitive conclusions, the study reported in this article provides firm support for several preliminary judgments. Empirical experience with the rules in the paint factory corroborates the findings of the research team. The methods have been used in the actual and satisfactory operation in the factory for several years now, and their use is currently being extended to other factories operated by the same company. The same method has also been adapted to several other production-scheduling situations in other companies and has satisfactorily passed “dry run” tests preliminary to actual installations in these situations. This report of findings is confined to the paint factory study, because this is the only one for which the data are publicly available at the present time.

Decision rules supplement, rather than displace, management judgment in scheduling production and employment. As such, they are of great value in helping management to:

Quantify and use the intangibles which are always present in the background of its thinking but which are incorporated only vaguely and sporadically in scheduling decisions. Make routine the comprehensive consideration of all factors relevant to scheduling decisions, thereby inhibiting judgment based on incomplete, obvious, or easily handled criteria. Fit each scheduling decision into its appropriate place in the historical series of decisions and through the feedback mechanism incorporated in the decision rules, automatically correct, prior forecasting errors.

In the case of the paint factory, for example, use of the decision rules permits regular monthly scheduling of production and employment to become a clerical function. Management attention can now be directed to refining cost estimates and periodically adjusting estimates to reflect hinges in costs resulting from modification of work flow and production process.

Besides this, management has more time to consider nonroutine factors and special situations that might provide reasons for modifying scheduling decisions computed from the mathematical rules. Anticipated changes in the raw material availability, in the supply of workers with the necessary skills, in customers’ procurement requirements, or in the character of the competitors’ service offering can get the attention they deserve from executives relieved of the burden of repetitive, complex scheduling decisions.

Management time is also free to develop ways and means of improving sales forecasting, with the knowledge that such gains can be fed directly into the decision rules and thus improve the efficiency.

But it would be shortsighted to think of the decision rules only in terms of the production setting of the paint factory. They can be modified to apply to other types of scheduling problems. The required changes are in the specific cost terms, not in the general structure of the rules. To be sure, the development of the rules in a different kind of plant requires careful study of the costs that are relevant to scheduling decisions, supported by explicit quantification of all cost elements. Subject to this limitation, however, the general technique is applicable to scheduling in any plant in which the relevant costs may be approximated by u-shaped curves.

Decision problems in areas outside production would also appear to be candidates for the application of mathematical decision rules of the type described. The scheduling of warehouse operations, of employment in retail stores, of certain classes of retail merchandise stocks, of working capital, and of some types of transportation operations, all appear to be fruitful areas for research. And with ingenuity management will undoubtedly discover still other applications in the future.

The broad implications of this study should be of interest not only to those persons directly concerned with production management but also to a wide managerial group.

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